

# A comment on power-law inflation with a dark radiation component

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Tram et al. 2016 recently pointed out in [1] that power-law inflation in presence of a dark radiation component may relieve the  $3.3\sigma$  tension which exists within standard  $\Lambda$ CDM between the determination of the local value of the Hubble constant by Riess et al. (2016) [2] and the value derived from CMB anisotropy data [3] by the *Planck* collaboration. In this comment, we simply point out that this interesting proposal does not help in solving the  $\sigma_8$  tension between the *Planck* data and, e.g., the weak lensing measurements. Moreover, when the latest constraints on the reionization optical depth obtained from *Planck* HFI data [4] are included in the analysis, the  $H_0$  tension reappears and this scenario loses appeal.

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## I. INTRODUCTION

The tension in the Hubble constant between the constraints coming from the *Planck* satellite [5] and [3] and the local measurements of Riess et al. [6] and [2] has recently gained statistical significance ( $3.3\sigma$ ) within the  $\Lambda$ CDM framework, especially when considering the new constraints on the reionization optical depth obtained with *Planck* HFI data [4].

Many proposals have been suggested to solve this tension (see for example [3, 5, 7–14]). Two possible extensions to the  $\Lambda$ CDM scenario have attracted significant attention. One is the possibility of a dark radiation component with  $N_{\text{eff}} > 3.046$ , which does not seem workable any more in light of the new *Planck* HFI constraint on the optical depth [4] for which  $N_{\text{eff}} = 2.91^{+0.39}_{-0.37}$  at 95% c.l. from Planck TTTEEE+SIMlow. The other extension is a dark energy equation of state different from  $w = -1$  (see [12]).

However, recently, the authors of [1] pointed out that, by considering a power-law inflation (hereafter PLI), introduced the first time by [15], in presence of a dark radiation component, the local value of the Hubble constant by Riess et al. 2016 [2] and the most recent CMB anisotropy data by the *Planck* collaboration [3] are in perfect agreement, provided  $\Delta N_{\text{eff}} = 0.62 \pm 0.17$  at 68% c.l..

Given the well-known correlations between parameters within  $\Lambda$ CDM, we consider in this comment the implication of this scenario (PLI with a free dark radiation) for the  $z = 0$  linear power normalisation,  $\sigma_8$  or clustering parameter. Indeed, within *Planck*-normalised  $\Lambda$ CDM, there is already a  $2\sigma$  tension with the weak lensing measurements of  $\sigma_8$  from the CFHTLenS survey [16, 17] and KiDS-450 [18]. We also assess the effect of adding the CMB polarization  $B$  modes constraint provided by the common analysis of *Planck*, BICEP2 and Keck Array [19], or the new determination on the reionization optical depth obtained with *Planck* HFI data [4] (which was not considered by [1]).

## II. METHOD

As a baseline, we will explore simultaneously 8 parameters of  $\Lambda$ CDM. These include the baryon and cold dark matter energy densities  $\Omega_b h^2$  and  $\Omega_c h^2$ , the ratio between the sound horizon and the angular diameter distance at decoupling  $\Theta_s$  and the reionization optical depth  $\tau$ . For the inflationary parameters, we consider the scalar spectral index,  $n_s$ , the amplitude of the primordial spectrum,  $A_s$ , and a contribution of primordial gravitational waves with a tensor-to-scalar ratio of amplitude  $r$  at the pivot scale  $k_0 = 0.05 h \text{Mpc}^{-1}$ . Finally, we also vary the effective number of relativistic degrees of freedom  $N_{\text{eff}}$ . We therefore include  $r$  and  $N_{\text{eff}}$  to the minimum 6 base  $\Lambda$ CDM parameters which provide an adequate fit to *Planck* data (assuming a flat Universe). These parameters are explored within the range of the conservative priors reported in Table I. In a second step,

TABLE I: External priors on the cosmological parameters assumed in this work.

Parameter . . . .	Prior
$\Omega_b h^2$ . . . . .	[0.005, 0.10]
$\Omega_{\text{cdm}} h^2$ . . . . .	[0.001, 0.99]
$\Theta_s$ . . . . .	[0.5, 10.0]
$\tau$ . . . . .	[0.01, 0.80]
$n_s$ . . . . .	[0.8, 1.20]
$\log[10^{10} A_s]$ . . . .	[2.0, 4.0]
$r$ . . . . .	[0., 0.50]
$N_{\text{eff}}$ . . . . .	[2.0, 5.0]

we repeat the same analysis considering the PLI model by imposing the following inflation consistency relationship [20]:

$$r = \frac{16n_{\text{T}}}{n_{\text{T}} - 2} \quad (1)$$

with

$$n_{\text{T}} = n_s - 1. \quad (2)$$

We find the constraints on these 8 parameters by combining several recent datasets. First of all, we call “PlanckTT + lowTEB” the full range of the 2015 temperature power spectrum ( $2 \leq \ell \leq 2500$ ) combined with the polarization power spectra in the multipoles range  $2 \leq \ell \leq 29$  provided by the *Planck* collaboration [21]. Secondly, when including the high multipoles *Planck* polarization data [21], we call this combination of datasets “PlanckTTTEEE + lowTEB” (which is considered less robust than the previous one by the *Planck* collaboration, at least for order  $1 \mu\text{K}^2$  wiggles relative to the plain  $\Lambda\text{CDM}$  polarisation spectra). Afterwards, when replacing the lowTEB dataset with a gaussian prior on the reionization optical depth  $\tau = 0.055 \pm 0.009$ , as obtained recently from *Planck* HFI data [4], we refer to it as “tau055”. Moreover, we consider the 2015 *Planck* measurements of the CMB lensing potential power spectrum  $C_\ell^{\phi\phi}$  [22], and we refer to this dataset as “lensing”. Finally, we add the CMB polarization *B* modes constraints provided by the 2014 common analysis of *Planck*, BICEP2 and Keck Array [19], and we refer to this dataset as “BKP”.

For definiteness, we have used the June 2016 version of the publicly available Monte-Carlo Markov Chain package *cosmomc* [23], with a convergence diagnostic based on the Gelman and Rubin statistic. This version, which we modified to include the PLI case, implements an efficient sampling of the posterior distribution using the fast/slow parameter decorrelations [24], and it includes the support for the *Planck* data release 2015 Likelihood Code [21] (see <http://cosmologist.info/cosmomc/>).

### III. RESULTS

The result of these explorations are given in Tables II and III where we report the constraints at 68% c.l. on the cosmological parameters, the two tables differing by considering or not the PLI specific constraint of eq. 1. By comparing the two Tables we see that imposing a PLI model affects the cosmological parameters in several ways. All the constraints that we will quote hereinafter there will be at 68% c.l., unless otherwise expressed.

First of all, PLI produces a shift towards higher values of the neutrino effective number  $N_{\text{eff}}$  reducing the error by a half. For Planck TT + lowTEB,  $N_{\text{eff}}$  changes from  $3.23^{+0.30}_{-0.36}$  to  $N_{\text{eff}} = 3.55^{+0.19}_{-0.17}$ . This shift corresponds to a higher  $H_0$  value,  $H_0 = 72.0^{+1.5}_{-1.1}$  Km/s/Mpc for the same combination of datasets, due to the degeneracy between these two parameters. This is illustrated in Fig. 1. This increase of the Hubble constant parameter solves the tension existing between the local measurements provided by Riess et al. 2016 [2], i.e.,  $H_0 = 73.00 \pm 1.75$  km/s/Mpc, and the value obtained from the *Planck* CMB anisotropy data [3], i.e.  $H_0 = 67.27 \pm 0.66$  km/s/Mpc in a  $\Lambda\text{CDM}$  framework, as argued in [1]. We note though that imposing the power-law inflation model, degrades somewhat the fit to the Planck TT + lowTEB data, producing a

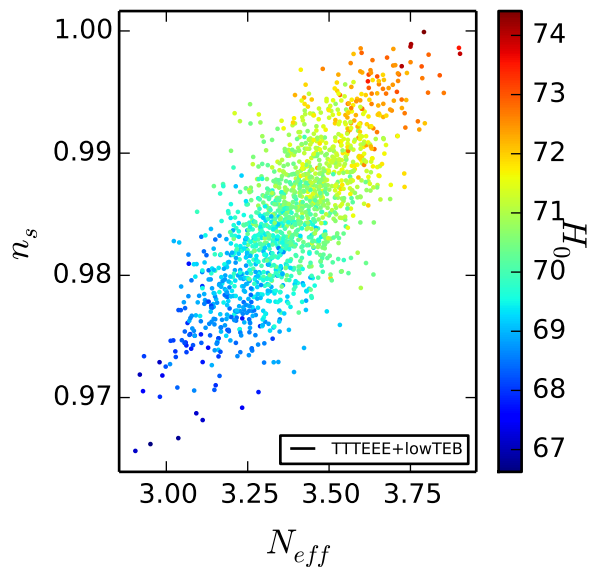


FIG. 1: Scattered 3D plot in the  $n_s$  versus  $N_{\text{eff}}$  plane, coloured by  $H_0$ , under the assumption of PLI.

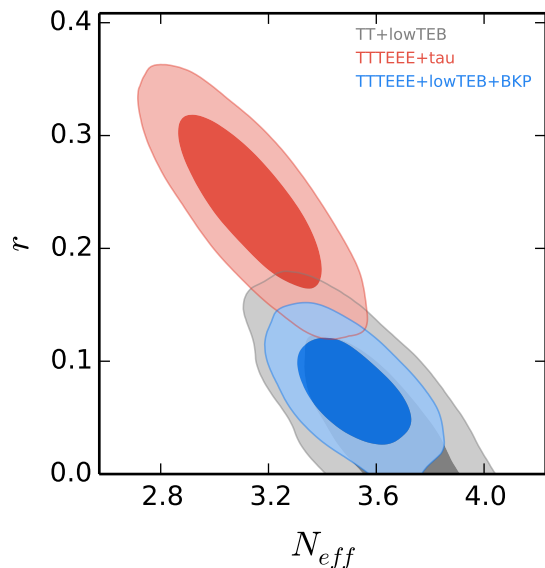


FIG. 2: Constraints at 68% and 95% confidence levels on the  $\tau$  vs  $N_{\text{eff}}$  plane under the assumption of PLI.

$$\Delta\chi^2 = 1.84.$$

Imposing PLI also leads to a higher value for the scalar spectral index  $n_s$ , that is only possible when  $N_{\text{eff}}$  is free to vary, since the two are positively correlated, as we can see in Fig. 1. This is also the reason why  $n_s$  goes down again and produces evidence for a non zero tensor-to-scalar ratio, as we can see in Fig. 2, when  $N_{\text{eff}}$  is further constrained by more data which shift it back towards the standard value.

Indeed the new constraints on the reionization opti-

	Planck TT + lowTEB	Planck TT + lowTEB + lensing	Planck TT + tau055	Planck TTTEEE + lowTEB	Planck TTTEEE + tau055	Planck TTTEEE + lowTEB + BKP
$\Omega_b h^2$	$0.02240 \pm 0.00037$	$0.02242^{+0.00035}_{-0.00041}$	$0.02182 \pm 0.00034$	$0.02219 \pm 0.00025$	$0.02194 \pm 0.00023$	$0.02219 \pm 0.00024$
$\Omega_c h^2$	$0.1214^{+0.0039}_{-0.0043}$	$0.1203 \pm 0.0039$	$0.1178 \pm 0.0041$	$0.1191 \pm 0.0031$	$0.1177 \pm 0.0031$	$0.1191 \pm 0.0031$
$\tau$	$0.082^{+0.021}_{-0.025}$	$0.072^{+0.018}_{-0.021}$	$0.0572 \pm 0.0089$	$0.076 \pm 0.018$	$0.0591 \pm 0.0088$	$0.077 \pm 0.018$
$n_S$	$0.974 \pm 0.016$	$0.976^{+0.014}_{-0.017}$	$0.946 \pm 0.016$	$0.963 \pm 0.010$	$0.9510 \pm 0.0095$	$0.9628 \pm 0.0096$
$\log(10^{10} A_S)$	$3.102^{+0.046}_{-0.052}$	$3.078 \pm 0.041$	$3.041 \pm 0.023$	$3.086 \pm 0.038$	$3.047 \pm 0.020$	$3.088 \pm 0.037$
$H_0$	$68.9^{+2.7}_{-3.2}$	$69.3^{+2.6}_{-3.1}$	$64.0^{+2.5}_{-2.9}$	$66.9 \pm 1.7$	$65.1 \pm 1.5$	$66.9 \pm 1.6$
$\sigma_8$	$0.837^{+0.022}_{-0.025}$	$0.824^{+0.018}_{-0.021}$	$0.809 \pm 0.014$	$0.827 \pm 0.018$	$0.809 \pm 0.012$	$0.828 \pm 0.017$
$N_{\text{eff}}$	$3.23^{+0.30}_{-0.36}$	$3.22^{+0.29}_{-0.34}$	$2.73^{+0.30}_{-0.34}$	$3.00 \pm 0.21$	$2.81 \pm 0.20$	$3.00 \pm 0.20$
$r$	$< 0.0529$	$< 0.0612$	$< 0.102$	$< 0.0487$	$< 0.103$	$0.036^{+0.011}_{-0.035}$

TABLE II: 68% c.l. constraints on cosmological parameters in our extended  $\Lambda\text{CDM}+r+N_{\text{eff}}$  scenario from different combinations of datasets.

	Planck TT + lowTEB	Planck TT + lowTEB + lensing	Planck TT + tau055	Planck TTTEEE + lowTEB	Planck TTTEEE + tau055	Planck TTTEEE + lowTEB + BKP
$\Omega_b h^2$	$0.02275 \pm 0.00024$	$0.02273 \pm 0.00024$	$0.02231 \pm 0.00026$	$0.02262 \pm 0.00020$	$0.02227 \pm 0.00019$	$0.02275 \pm 0.00016$
$\Omega_c h^2$	$0.1240 \pm 0.0036$	$0.1226 \pm 0.0033$	$0.1222 \pm 0.0037$	$0.1231 \pm 0.0028$	$0.1211 \pm 0.0030$	$0.1243 \pm 0.0028$
$\tau$	$0.095 \pm 0.020$	$0.085 \pm 0.015$	$0.0602 \pm 0.0088$	$0.092 \pm 0.017$	$0.0620 \pm 0.0087$	$0.100 \pm 0.017$
$n_S$	$0.9918^{+0.0072}_{-0.0035}$	$0.9911^{+0.0074}_{-0.0038}$	$0.9735 \pm 0.0083$	$0.9849 \pm 0.0057$	$0.9695 \pm 0.0062$	$0.9907^{+0.0042}_{-0.0034}$
$\log(10^{10} A_S)$	$3.134 \pm 0.039$	$3.111 \pm 0.028$	$3.060 \pm 0.020$	$3.126 \pm 0.035$	$3.062 \pm 0.020$	$3.146 \pm 0.034$
$H_0$	$72.0^{+1.5}_{-1.1}$	$72.0^{+1.5}_{-1.1}$	$68.7 \pm 1.7$	$70.3 \pm 1.1$	$67.8 \pm 1.2$	$71.32 \pm 0.84$
$\sigma_8$	$0.854 \pm 0.018$	$0.841 \pm 0.012$	$0.822 \pm 0.013$	$0.850 \pm 0.017$	$0.821 \pm 0.011$	$0.861 \pm 0.016$
$N_{\text{eff}}$	$3.55^{+0.19}_{-0.17}$	$3.51 \pm 0.17$	$3.24 \pm 0.22$	$3.39 \pm 0.15$	$3.14 \pm 0.17$	$3.51 \pm 0.13$
$r$	$0.065^{+0.020}_{-0.062}$	$0.071^{+0.027}_{-0.060}$	$0.209 \pm 0.064$	$0.120 \pm 0.045$	$0.240 \pm 0.048$	$0.074^{+0.027}_{-0.033}$

TABLE III: 68% c.l. constraints on cosmological parameters in our extended  $\Lambda\text{CDM}+r+N_{\text{eff}}$  scenario from different combinations of datasets with a power-law inflation.

cal depth from *Planck* HFI data [4] reinstates compatibility with the standard value of 3.046 for the neutrino effective number  $N_{\text{eff}}$  which restores the tension on  $H_0$ . Specifically, we find  $N_{\text{eff}} = 3.24 \pm 0.22$  and  $H_0 = 68.7 \pm 1.7$  Km/s/Mpc for Planck TT + tau055, and  $N_{\text{eff}} = 3.14 \pm 0.17$  and  $H_0 = 67.8 \pm 1.2$  Km/s/Mpc for Planck TTTEEE + tau055. Moreover, an evidence on  $r$  different from zero at more than  $3\sigma$  appears: we find  $r = 0.209 \pm 0.064$  for Planck TT + tau055 and

$r = 0.240 \pm 0.048$  for Planck TTTEEE + tau055. In this case also, when imposing PLI, the fit to the Planck TT + tau055 data gets worse, producing a  $\Delta\chi^2 = 5.32$  for one less degree of freedom.

For the same reason, the clustering parameter  $\sigma_8$  moves towards higher values when imposing PLI, since it is positively correlated with the Hubble constant, see Fig. 3. Numerically, it moves from  $\sigma_8 = 0.837^{+0.022}_{-0.025}$  to  $\sigma_8 = 0.854 \pm 0.018$  when imposing a power-law inflation

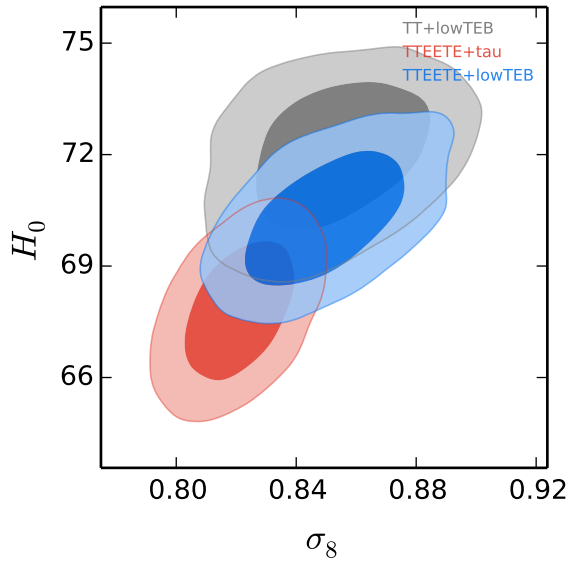


FIG. 3: Constraints at 68% and 95% confidence levels on the  $H_0$  vs  $\sigma_8$  plane under the assumption of PLI.

for Planck TT + lowTEB. This enhancement of the  $\sigma_8$  value therefore increases notably the tension between the *Planck* data and the weak lensing measurements from the CFHTLenS survey [16, 17] and KiDS-450 [18].

Finally, when we include the CMB polarization  $B$  modes dataset provided by the common analysis of *Planck*, BICEP2 and the Keck Array [19], a value of  $r$  different from zero at more than  $2\sigma$  appears, i.e.,  $r = 0.074^{+0.027}_{-0.033}$  for Planck TTTEEE + lowTEB + BKP

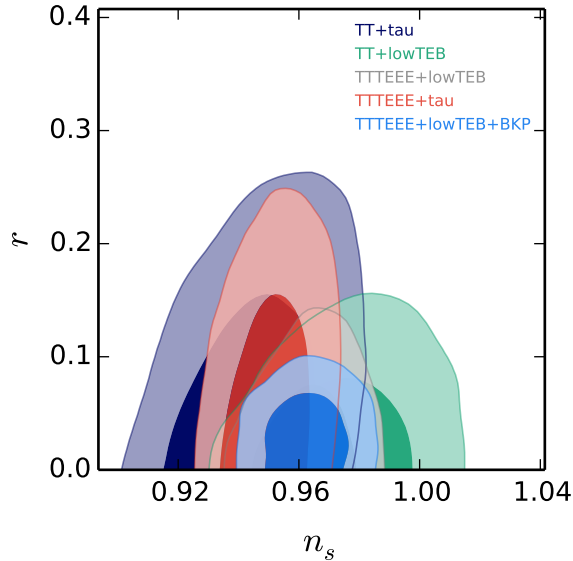


FIG. 4: Constraints at 68% and 95% confidence levels on the  $r$  vs  $n_s$  plane, without considering the PLI.

in the power-law inflation, and the agreement between  $H_0$  from Riess et al. 2016 [2] and the *Planck* data is confirmed. Here again, the fit to the combination of datasets Planck TTTEEE + lowTEB + BKP gets worse when imposing the power-law inflation, producing a  $\Delta\chi^2 = 9.14$  for one less degree of freedom. This evidence for  $r$  different from zero is totally absent without imposing the power-law inflation, as we can see in Fig. 4.

#### IV. CONCLUSIONS

Recently, the authors of [1] pointed out that by considering a power law inflation model and a free dark radiation component, the local measurements of the Hubble constant provided by Riess et al. 2016 [2], i.e.,  $H_0 = 73.00 \pm 1.75$  km/s/Mpc at 68% cl, is in perfect agreement with the value that follows when analysing the *Planck* CMB anisotropy data [3], inducing a  $\Delta N_{\text{eff}} = 0.62 \pm 0.17$  at 68% cl.

In this comment, we confront that scenario (PLI+dark radiation) with more data than initially considered. As noted in the previous section, the Hubble constant and the clustering parameter are positively correlated, therefore a higher  $H_0$  value corresponds to an increased value of  $\sigma_8$ . When the  $H_0$  tension subsides, this degeneracy produces a shift of the mean value of the clustering parameter which exacerbates the tension with the weak lensing measurements from the CFHTLenS survey [16, 17] and KiDS-450 [18].

We then considered the implication of adding the CMB polarization  $B$  modes dataset provided by the common analysis of *Planck*, BICEP2 and Keck Array [19]. In this case, interestingly, a value of  $r$  different from zero at more than  $2\sigma$  is preferred,  $r = 0.074^{+0.027}_{-0.033}$  at 68% cl for Planck TTTEEE + lowTEB + BKP (in the power-law inflation case), while the agreement between  $H_0$  from Riess et al. 2016 [2] and the *Planck* data is maintained. However, when considering the new constraints on the reionization optical depth from Planck HFI data [4], the lower value preferred by that data on the neutrino effective number  $N_{\text{eff}}$  restores the tension on  $H_0$  between the datasets and the standard value for  $N_{\text{eff}}$  is recovered. The most recent data (given the  $\Delta\chi^2$ ) therefore does not really lend support to the power-law inflation model with dark radiation.

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- [1] T. Tram, R. Vallance and V. Vennin, arXiv:1606.09199 [astro-ph.CO].
- [2] A. G. Riess *et al.*, arXiv:1604.01424 [astro-ph.CO].
- [3] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO].
- [4] N. Aghanim *et al.* [Planck Collaboration], arXiv:1605.02985 [astro-ph.CO].
- [5] P. A. R. Ade *et al.* [Planck Collaboration], *Astron. Astrophys.* **571** (2014) A16 doi:10.1051/0004-6361/201321591 [arXiv:1303.5076 [astro-ph.CO]].
- [6] A. G. Riess *et al.*, *Astrophys. J.* **730** (2011) 119 Erratum: [*Astrophys. J.* **732** (2011) 129] doi:10.1088/0004-637X/732/2/129, 10.1088/0004-637X/730/2/119 [arXiv:1103.2976 [astro-ph.CO]].
- [7] A. Heavens, R. Jimenez and L. Verde, *Phys. Rev. Lett.* **113** (2014) no.24, 241302 doi:10.1103/PhysRevLett.113.241302 [arXiv:1409.6217 [astro-ph.CO]]; E. Di Valentino, E. Giusarma, O. Mena, A. Melchiorri and J. Silk, arXiv:1511.00975 [astro-ph.CO]; E. Di Valentino, E. Giusarma, M. Lattanzi, O. Mena, A. Melchiorri and J. Silk, *Phys. Lett. B* **752** (2016) 182 doi:10.1016/j.physletb.2015.11.025 [arXiv:1507.08665 [astro-ph.CO]]; M. Archidiacono, E. Giusarma, S. Hannestad and O. Mena, *Adv. High Energy Phys.* **2013** (2013) 191047 doi:10.1155/2013/191047 [arXiv:1307.0637 [astro-ph.CO]].
- [8] V. Salvatelli, A. Marchini, L. Lopez-Honorez and O. Mena, *Phys. Rev. D* **88** (2013) no.2, 023531 doi:10.1103/PhysRevD.88.023531 [arXiv:1304.7119 [astro-ph.CO]]; A. A. Costa, X. D. Xu, B. Wang, E. G. M. Ferreira and E. Abdalla, *Phys. Rev. D* **89** (2014) no.10, 103531 doi:10.1103/PhysRevD.89.103531 [arXiv:1311.7380 [astro-ph.CO]]; A. Pourtsidou, C. Skordis and E. J. Copeland, *Phys. Rev. D* **88** (2013) no.8, 083505 doi:10.1103/PhysRevD.88.083505 [arXiv:1307.0458 [astro-ph.CO]].
- [9] K. Ichiki, C. M. Yoo and M. Oguri, *Phys. Rev. D* **93** (2016) no.2, 023529 doi:10.1103/PhysRevD.93.023529 [arXiv:1509.04342 [astro-ph.CO]].
- [10] I. Ben-Dayan, R. Durrer, G. Marozzi and D. J. Schwarz, *Phys. Rev. Lett.* **112**, 221301 (2014) doi:10.1103/PhysRevLett.112.221301 [arXiv:1401.7973 [astro-ph.CO]].
- [11] V. Marra, L. Amendola, I. Sawicki and W. Valkenburg, *Phys. Rev. Lett.* **110** (2013) no.24, 241305 doi:10.1103/PhysRevLett.110.241305 [arXiv:1303.3121 [astro-ph.CO]]; I. Odderskov, S. Hannestad and T. Haugbille, *JCAP* **1410** (2014) no.10, 028 doi:10.1088/1475-7516/2014/10/028 [arXiv:1407.7364 [astro-ph.CO]]; I. Odderskov, S. M. Koksang and S. Hannestad, *JCAP* **1602**, no. 02, 001 (2016) doi:10.1088/1475-7516/2016/02/001 [arXiv:1601.07356 [astro-ph.CO]].
- [12] E. Di Valentino, A. Melchiorri and J. Silk, arXiv:1606.00634 [astro-ph.CO].
- [13] J. L. Bernal, L. Verde and A. G. Riess, arXiv:1607.05617 [astro-ph.CO].
- [14] P. Ko and Y. Tang, arXiv:1608.01083 [hep-ph].
- [15] F. Lucchin and S. Matarrese, *Phys. Rev. D* **32**, 1316 (1985). doi:10.1103/PhysRevD.32.1316
- [16] C. Heymans *et al.*, *Mon. Not. Roy. Astron. Soc.* **427**, 146 (2012) [arXiv:1210.0032 [astro-ph.CO]].
- [17] T. Erben *et al.*, *Mon. Not. Roy. Astron. Soc.* **433**, 2545 (2013) [arXiv:1210.8156 [astro-ph.CO]].
- [18] H. Hildebrandt *et al.*, arXiv:1606.05338 [astro-ph.CO].
- [19] P. A. R. Ade *et al.* [BICEP2 and Planck Collaborations], *Phys. Rev. Lett.* **114** (2015) 10, 101301 [arXiv:1502.00612 [astro-ph.CO]].
- [20] J. Martin, C. Ringeval and V. Vennin, *Phys. Dark Univ.* **5-6**, 75 (2014) doi:10.1016/j.dark.2014.01.003 [arXiv:1303.3787 [astro-ph.CO]].
- [21] N. Aghanim *et al.* [Planck Collaboration], arXiv:1507.02704 [astro-ph.CO].
- [22] P. A. R. Ade *et al.* [Planck Collaboration], doi:10.1051/0004-6361/201525941 arXiv:1502.01591 [astro-ph.CO].
- [23] A. Lewis and S. Bridle, *Phys. Rev. D* **66**, 103511 (2002) [astro-ph/0205436].
- [24] A. Lewis, *Phys. Rev. D* **87**, no. 10, 103529 (2013) [arXiv:1304.4473 [astro-ph.CO]].